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Correlated quasiskyrmions as alpha-particles

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Abstract. By replacing quasiparticles by quasiskyrmions, we calculate the binding energy of the alpha-particle using the cluster expansion method.

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1 Introduction

Following an old idea of Skyrme [1] baryons emerge as chiral solitons in a nonlinear sigma-model when a term with four derivatives is added to the Lagrangian. Witten [2] has shown the connection between Skyrme's model and QCD in the large-N_c limit and proved Skyrme's suggestion of identifying the integrated topological current W_μ with the baryon number. Static properties of the low-lying baryons [3] as well as the skyrmion-skyrmion interaction [4] have been worked out in detail. These calculations are based on the most general *ansatz* for $SU(2)$ field of the underlying chiral field which arises due to the principle of maximal symmetry

$$U(\mathbf{r}) = e^{i\boldsymbol{\tau}\cdot\hat{n}\theta(\mathbf{r})}\,, \tag{1}$$

where $\hat{n}(r)$ is some unit vector and $\theta(\mathbf{r})$ is the chiral field related to the σ and $\boldsymbol{\pi}$ degrees of freedom. In the case of the simple hedgehog, $\hat{n} = \mathbf{r}\backslash|\mathbf{r}|$ and $\theta(\mathbf{r}) = \theta(|\mathbf{r}|)$, *i.e.*, the isospin points radially in space and the chiral field depends on one variable, the distance r. Atiyah and Manton [5] have derived an analytic form of the shape function from the instanton ansatz for one and two skyrmions:

$$\theta_1(r) = \pi\left[1 - \left(1 + \frac{\lambda^2}{r^2}\right)^{-0.5}\right], \tag{2}$$

$$\theta_2(r) = 2\pi\left[1 - \left(1 + \frac{\lambda^2}{r^2}\right)^{-0.5}\right], \tag{3}$$

where λ is equal to $\frac{L}{eF_\pi}$, L being a variational parameter and e, F_π are constants. Note that the instanton ansatz (and this shape function) is only an approximation.

Applying Skyrme's model to larger nuclei and to nuclear matter is an interesting proposition. Recently, there has been some progress in understanding the structure of multi-skyrmions [6,7]. But, unfortunately there is one piece lacking in all these works, at least to our knowledge, which we next bring to the attention of the reader. The correlation effect in multi-skyrmions and nuclear matter calculations has been neglected so far, and the cluster methods have not been applied to the skyrmions. From our point of view this is not consistent. It is well known that even very straightforward approximations based on the inclusion of two-body correlations in various ways, lead to binding energies accurate by comparison with other models [8]. Investigation of the *approximate* behavior of correlated skyrmions, is a motivation for this work. Although the instanton ansatz is a bad approximation to the true minimum energy solution which only has axial symmetry for $B = 2$ [9], nonetheless it is sufficiently accurate for our purposes.

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2 Cluster expansion and quasiskyrmions

There have been many attempts lately to calculated the properties of light atomic nuclei, as for example ^{4}He, by the various techniques of modern microscopic quantum many-body theory. The approach employed here is based on the very well-known variational cluster expansion technique of Clark and Westhaus [10]. Particles are replaced by correlated quasiparticles (hereafter quasiskyrmions) and the use of a very simplified Jastrow parametrization of the two-body correlation function:

$$f_J(r) = 1 - D\exp\left(\frac{-r^2}{C^2}\right), \tag{4}$$

where D and C are variational parameters. Moreover, since the cluster expansion is written as a power of the correlation function, for the range of inter-skyrmion potentials considered here, calculations are truncated at the lowest possible two-body level, such that

$$E = \sum_{i>j} \langle ij| \frac{\hbar^2}{m}[\nabla f(r)]^2 + f^2(r)V(r)|ij\rangle + T_{\mathrm{F}}\,, \qquad (5)$$

where T_{F}, $V(r)$ are, respectively, the Fermi kinetic energy and a nonsingular effective potential applied between the quasiparticles. The most general form of the Slater determinant $|ij\rangle$ is

$$\begin{aligned}|ij\rangle = &\sum (1/2\sigma_1 l_1 m_{l_1}|j_1\, m_1)(1/2\sigma_2 l_2 m_{l_2}|j_2\, m_2)\\ &\times (1/2\sigma_1 1/2\sigma_2|SM_S)(1/2\tau_1 1/2\tau_2|T\, M_T)\\ &\times \Psi_{n_1 l_1 m_{l_1}}(r_1)\Psi_{n_2 l_2 m_{l_2}}(r_2)\chi_{SM_S}\tau_{TM_T}\,, \qquad (6)\end{aligned}$$

where

$$\begin{aligned}&[\Psi_{n_1 l_1}(r_1)\Psi_{n_2 l_2}(r_2)]^\lambda =\\ &\sum_{nlNL} \langle nlNL\lambda|\, n_1 l_1 n_2 l_2 \lambda\rangle [\Psi_{nl}(r_{\mathrm{int}})\Psi_{NL}(R_{CM})]^\lambda; \qquad (7)\end{aligned}$$

the coefficients in eq. (7) are the so-called Brody-Moshinsky brackets and $\Psi(r)$ is the variational wave function in Jastrow-theory, which is taken as

$$\Psi_J(r_{\mathrm{int}}) = \prod_{i>j} f_J(r_{\mathrm{int}})\Phi\,, \qquad (8)$$

where Φ denotes $\Phi(r_{\mathrm{int}})$. Since $\int \Psi^2_{NL}(R_{CM})\mathrm{d}^3R_{CM} = 1$, we separate the two-particle cluster's centre of mass. Then we assume a skyrmion density which is taken as the square of wave function $\Psi_J(r_{\mathrm{int}})$ and define

$$\Phi^*\Phi = 4\pi r^2 W_0(r)\,, \qquad (9)$$

where $W_0(r)$ is the topological charge density [3]:

$$W_0(r) = \frac{-1}{2\pi^2}\frac{\sin^2\theta(r)}{r^2}\frac{\mathrm{d}\theta(r)}{\mathrm{d}r}\,. \qquad (10)$$

We plug in this expression the approximate Atiyah-Manton baryon density profile $\theta(r)$ (for the two-skyrmion system) instead of the two-skyrmion wave function. This procedure can be taken as an *axiom*. The merit for this choice lies in the fact that the derived wave function $\Psi_J(r)$ is normalizable. The baryon number fractionalisation of the model is left for a future publication.

3 Calculations and results

The procedure we develop in here is intermediate between the more formal Skyrme model studies and the traditional nuclear-physics approaches. The aim has been to replace the skyrmion-skyrmion interaction by phenomenological NN potentials and approach the data from that standpoint. We represent a two-nucleon cluster by a matter density of correlated skyrmions of baryon number 2 with approximate two-skyrmion chiral angle and use effective *nuclear* potentials B1 [11], S3 [12] and M.T [13] for the inter-skyrmion interaction. Our procedure may seem awkward since we have used a skyrmion profile for the square of the wave function and effective nuclear potential for the interaction, a scheme not fully justified. However, we have done so only for comparing the obtained density distribution from Skyrme's picture with that of other nuclear theory methods. (The phenomenology of an actual 4-skyrmions particle is left for a future publication.)

Table 1. Calculated values of the binding energies of the alpha-particle (third row) and comparison with two nucleonic systems calculations (first and second row). All of these magnitudes are given in MeV.

Potentials	B1	S3	M.T
Jastrow	−36.44	−24.29	−29.48
LTICC	−37.8	−25.29	−26.77
Correlated quasiskyrmions	−25.43	−19.35	−22.42

In a recent publication Irwin [7], using the zero-mode quantization, has shown that the ground state for the $B = 4$ skyrmion has spin and isospin zero and positive parity, a feature which we used in our calculations. For obtaining T_{F}, we assumed that the Fermi kinetic energy is associated with the vibration of the $B = 1$ skyrmions and that noninteracting skyrmions oscillate harmonically away from their equilbrium position parmetrized by constant L. In 3-dimensional space the quantization of the Skyrme Lagrangian modes leads to $E = (n + \frac{1}{2})\hbar\omega$ [14]. Then a simple calculation for approximating the Fermi kinetic energy yields

$$T_{\mathrm{F}} = 0.75\frac{\hbar^2 L^{-2}}{m} \qquad (11)$$

(where $\frac{\hbar^2}{m}$ is equal to 41.5 MeV $(\mathrm{fm})^2$). This is also in agreement with the work of Irwin which shows that the vibrational states are important and have an energy of the same order as the pure rotational state.

We proceed to calculate the binding energy of the alpha-particle by means of a variational principle implying minimization with respect to the variational parameters: L, C, and D. The results are shown in table 1, where we compare the binding energies with two traditional nuclear theory methods: i) an ancient method which consists of a simple variational calculations for a trial Jastrow wave function and ii) a more accurate method which assumes that the correlated wave function of a many-particle system is decomposed in terms of the amplitude for exciting clusters of a finite number of particles [15]. It is clear that the approach introduced in this work is worst than the other methods, since the skyrmions are classical approximations describing the nucleons. It is apparent that the behavior of the skyrmion profile is not bad for the strong S3 and M.T potentials, while for the soft B1 potential, since the correlation effects tend to zero, the resutls are poor. We conclude from the analysis that, if the skyrmion profile is to represent the nucleon density as in the approach above, the internuclear potential between correlated skyrmions has to be strong.

In summary: skyrmions, which are basically free at large separations, become strongly correlated when they are brought together. Therefore cluster methods should be employed in the study of skyrmion physics, *i.e.*, in the description of nuclear physics in the language of the large-N_c QCD.

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